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Ex Post Private Information and Monopolistic Screening

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Abstract

This paper investigates how additional ex post private information by the agent affects the equilibrium outcome of the monopolistic screening model. In general, the principal always weakly benefits when the agent receives additional private information after the contracting stage. Instead, both the agent's equilibrium payoffs and allocative efficiency may, due to the principal's concerns about information rents, increase or decrease. Moreover, we obtain the result that optimal contracts may involve lying off–the–equilibrium path and show that this exacerbates bunching in the monopolistic screening problem.

KEYWORDS: monopolistic screening, sequential screening, adverse selection, ex post private information

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1 Introduction

The monopolistic screening model has proven itself an extremely versatile modeling tool to study the effect of private information. It has advanced our understanding of asymmetric information in taxation, insurance, price–discrimination, regulation and many other economic settings. The model presumes that all private information is obtained ex ante, before the economic parties meet. Clearly, this is a simplifying abstraction. In practice, additional, decision relevant information is frequently revealed during the course of the relationship. Specific examples include procurement of goods or services where the contractor obtains more precise cost information as the date of production approaches, health insurance where insureds receive additional information about their health status, or a manager who, in his fiduciary role, receives private information about the viability of the firm. It is therefore important to understand if the insights from the monopolistic screening model carry over to contexts where the agent receives such ex post private information. In this paper, we are especially interested in how ex post information affects both allocative efficiency and equilibrium payoffs. This is likely to have important implications for the regulation of screening practices and information acquisition incentives.

The issue is non–trivial, as exemplified by the following two contradicting intuitions which the existing literature on screening offers. First, one may argue that additional ex post private information increases the overall degree of asymmetric information and, hence, we should expect a more intense trade–off between rents and efficiency that characterizes the monopolistic screening model. This indicates larger information rents to the agent and, as a result, a worsening of allocative efficiency. Second, Esö and Szentes (2007) show that a principal can extract the agent's ex post private information costlessly in the sense that she obtains the same payoff as if the additional information was publicly available. This result suggests that ex post private information does not worsen the trade–off between rent and efficiency.

Faced with these contradicting intuitions, this paper explicitly compares the equilibrium outcomes of the standard screening model without ex post private information to a screening model with ex post private information. It shows that, in general, ex post private information indeed has ambiguous effects. The only unambiguous result is that the principal is always weakly better off. This follows from a straightforward replication argument: the principal can always offer the second best contract which ignores the additional information. In this way, she can guarantee herself the same payoff as in the absence of ex post information.

In contrast, the effect on the agent's equilibrium payoff and, more importantly, on efficiency is ambiguous. Both information rents and overall efficiency may increase or decrease due to the presence of additional ex post private information. The reason behind the ambiguity is the trade–off between efficiency and rent extraction: instead of maximizing total surplus, the principal maximizes the share of the surplus that she can extract. Intuitively, the principal may use the additional information to better tailor the allocation to actual costs so as to increase total surplus and then extract a larger share of the larger surplus. In this case, efficiency unambiguously goes up. Our key insight, however, is that this need not be the case. We explicitly show that the principal may maximize his share of the surplus by actually lowering the total surplus and then extract a larger share of the overall lower surplus. In this sense, rent extraction concerns may prevail *at the expense* of efficiency concerns.

The extent to which ex post information affects efficiency and rents has important economic implications. First, a natural question that arises is whether a government should regulate the design of contracts and, in particular, allow screening for additional information. Our analysis identifies cases in which the simple policy instrument of prohibiting contracts to use ex post information is welfare enhancing. For instance, Courty and Li (2000) argue that contracts which condition on ex post information are often implemented by menus that include refund or exit options. Thus, prohibiting refunds may be welfare enhancing.

Moreover, the effect of ex post information on the distribution of rents is also important to understand the agent's incentives for obtaining such information. Indeed, an agent will acquire ex post information only if it increases his rents. Given that the agent may actually lose from obtaining additional information, the agent may try to device strategies that prevent him from obtaining any ex post private information and stay ignorant (see also Kessler, 1998, for a similar effect in a principal agent model but without sequential screening). However, the question of information acquisition is more involved than a straightforward comparison of equilibrium outcomes with and without ex post private information. When information acquisition becomes a strategic decision, both the principal's and the agent's incentives are changed. A comprehensive analysis therefore goes well beyond the scope of the current paper, and is provided in our companion paper (Krähmer and Strausz, 2008).[1]

[1]Because the current paper focuses on the revelation of ex post private information rather than its active acquisition, it is only indirectly related to the literature on the acquisition and use of private information (e.g. Lewis and Sappington, 1997, Cremer and Khalil, 1992,

The monopolistic screening model with additional ex post private information has been analyzed in Courty and Li (2000), who coined the term ”sequential screening”. The idea is that after a first screening at the contract stage, the principal screens the agent again after he receives additional private information. Although Courty and Li characterize the structure of optimal sequential screening contracts, their general model is too intractable to study the effects of additional private information on equilibrium payoffs and allocative efficiency. We therefore study a simpler discrete version of a sequential screening model which is still rich enough to exhibit all the salient features. Although our results are broadly consistent with those of Courty and Li (2000), there are some notable differences. Unlike Courty and Li (2000), we allow for the case that the support of the second observation depends on the first. This feature leads to the qualitative difference that optimal contracts may involve lying off the equilibrium path. Off–the–equilibrium–path–lying occurs whenever the degree of ex post private information is small. In this case, ex post information does not affect the allocation of the inefficient type; agents with different ex post information are bunched together and receive the same allocation. Interestingly, this allocation coincides with the allocation that is offered under the standard contract without sequential screening. This reveals that the monopolistic screening model is robust with respect to small degrees of ex post information.

At first sight our results seem to contradict those of Esö and Szentes (2007), who also study the effect of additional ex post private information. These authors however use a different benchmark than we do. In particular, they compare the outcome when the ex post private information is the agent's private information to the outcome when the ex post information is public. In the latter case, the ex post information is directly contractible, in the former, the principal has to set incentives for truthful revelation. Esö and Szentes show that the outcomes of the two models are the same. In line with us, they therefore conclude that the principal always gain from providing the agent with ex post private information. However, as our results show, it would be wrong to deduce that also the agent (weakly) gains from his private information. Indeed, our results show that this is generally not the case. In contrast to Esö and Szentes, we compare the outcome when there is no ex post information to the outcome when the agent receives ex post private information. We show that ex post private information may reduce the agent's information rents and reduce overall efficiency. In the application of Esö and Szentes, where the principal decides to reveal private information to the agent, this means that

Cremer et al., 1998a, 1998b).

the principal may need to *force* the agent rather than *allow* the agent to receive the information. Moreover, the principal's release of information may lower overall efficiency.

A further related paper is Dai et al. (2006). They study a sequential screening procurement model in which ex ante the agent might be an "expert" or a "non-expert" who, after contracting, receives more or less precise information about his true production costs. Experts and non-experts face the same expected production costs ex ante which can imply that the optimal contract distorts the non-expert's production quantity upwardly. This cannot happen in our context, because we assume that expected costs are different across agent types. Moreover, Dai et al.'s analysis focuses on comparatives statics of the principal's profit and on the agent's incentives to become an expert in the first place. We, instead, focus on how additional ex post private information per se changes the agent's rent and overall efficiency.

The paper is organized as follows. In section 2, we present the model. In section 3, we discuss the benchmark case of the standard model in which there is no ex post private information. Section 4 describes the contracting problem with ex post private information whose solution we present in section 5. In section 6, we discuss the effects of ex post private information, and section 7 concludes.

2 The Model

Consider an agent who can realize an indivisible project that has a value of $V > 0$ to the principal. The overall cost of the agent is uncertain and depends on two states, s_1 and s_2. In particular, overall costs are

$$c(s_1, s_2) = \alpha s_1 + (1 - \alpha)s_2.$$

We consider the simplest case with binary states.[2] That is, let $s_t \in \{s_{tl}, s_{th}\}$ and define $\Delta s_t = s_{th} - s_{tl} > 0$. The state s_{1l} occurs with probability γ, and the state s_{1h} occurs with probability $1 - \gamma$. Let μ_i represent the conditional probability that the state s_2 is low given the state s_{1i}:

$$\mu_i \equiv Pr\{s_2 = s_{2l} \mid s_1 = s_{1i}\}.$$

All of this is common knowledge to the principal and the agent.

[2]The restriction to binary states is mainly helpful to keep bunching issues tractable. The binary setup is nevertheless rich enough to demonstrate all important effects.

Throughout we assume that the agent *privately* observes the state s_1 perfectly *before* contracting takes place. Concerning the observability of the state s_2 we will distinguish between two cases. In the benchmark case, neither the principal nor the agent observes s_2. As a result, the model exhibits only ex ante asymmetric information. It then coincides with the standard monopolistic screening model of Baron and Myerson (1982) with the inconsequential difference that the agent's private information is not fully informative about the cost of the project.

Our main interest is in contrasting the outcome of this standard setup to one where the agent receives information about the state s_2 *after* contracting has been signed but before it is implemented. We say that under such an information structure the agent receives ex post private information. For convenience, we assume that the agent observes s_2 perfectly. The parameter $\alpha \in [0, 1]$ measures the relative importance of the two states on total costs, and thus captures the relative importance of ex post information. For a given value of α, we will identify the differences in the equilibrium outcomes between the benchmark case and the model with ex post private information.

Since states are binary there are four possible realizations of total cost: c_{ll}, c_{lh}, c_{hl}, c_{hh}, where

$$c_{ij} = \alpha s_{1i} + (1 - \alpha)s_{2j}.$$

Due to $s_{th} > s_{tl}$ we have the ordering $c_{ll} \leq c_{lh} \leq c_{hh}$ and $c_{ll} \leq c_{hl} \leq c_{hh}$. The ordering between c_{lh} and c_{hl} depends on the parameter α. Let

$$\bar{\alpha} = \frac{\Delta s_2}{\Delta s_1 + \Delta s_2}.$$

Then

$$\alpha < \bar{\alpha} \Rightarrow \; c_{ll} < c_{hl} < c_{lh} < c_{hh},$$

and

$$\alpha > \bar{\alpha} \Rightarrow \; c_{ll} < c_{lh} < c_{hl} < c_{hh}.$$

We allow for correlation between the states s_1 and s_2. Without loss of generality, we assume $\mu_l \geq \mu_h$ so that an agent who observes the state s_{1l} expects lower overall costs. This follows from

$$\begin{aligned} c_l \equiv E\{c|s_1 = s_{1l}\} &= \mu_l c_{ll} + (1 - \mu_l)c_{lh} \\ &\leq \mu_l c_{hl} + (1 - \mu_l)c_{hh} \\ &\leq \mu_h c_{hl} + (1 - \mu_h)c_{hh} \\ &= E\{c|s_1 = s_{1h}\} \equiv c_h, \end{aligned}$$

where the first inequality is due to $c_{lj} \leq c_{hj}$, and the inequality in the third line follows since $c_{hl} \leq c_{hh}$ and $\mu_l \geq \mu_h$. In what follows, we refer to the agent who observes the state s_{1l} (resp. s_{1h}) as the low cost or efficient type (resp. the high cost or inefficient type). Next, we turn to the principal's contracting problem.

3 No Ex Post Private Information

We begin with the benchmark case in which the agent observes s_1 but not s_2. The setup then conforms to Baron and Myerson (1982) with two types, where an agent who has observed s_{1i} expects a cost of c_i. This benchmark is equivalent to the case in which the agent does observe s_2 after contracting but the contract can only condition on the agent's ex ante information s_1. The outcome of this familiar setup serves as a point of reference for evaluating the effects of ex post private information.

From Baron and Myerson (1982) it follows that, without ex post private information, the optimal contract is a direct mechanism (t_l, q_l, t_h, q_h) that gives the agent an incentive to report his cost type truthfully. In particular, it solves the following problem:

$$\max_{(t,q)} \; W = \gamma(q_l V - t_l) + (1-\gamma)(q_h V - t_h) \tag{1}$$

$$\text{s.t.} \quad t_i - c_i q_i \geq t_j - c_i q_j \quad \text{for all } i, j = 1, 2; \tag{2}$$

$$t_i - c_i q_i \geq 0 \quad \text{for all } i = 1, 2. \tag{3}$$

where the constraints in (2) represent the incentive constraints and the constraints in (3) express the agent's participation constraints.

As is standard, only the incentive constraint of the efficient type s_{1l} and the individual rationality constraint of the inefficient type s_{1h} are binding at the optimum. Solving for the optimal direct mechanism yields

$$q_l^{sb} = \begin{cases} 1 & \text{if } V \geq C_l \equiv c_l \\ 0 & \text{if } V < C_l; \end{cases} \qquad q_h^{sb} = \begin{cases} 1 & \text{if } V \geq C_h \equiv c_h + \frac{\gamma}{1-\gamma}(c_h - c_l) \\ 0 & \text{if } V < C_h. \end{cases}$$

Transfers are

$$t_l = t_l^{sb} \equiv c_l + (c_h - c_l) q_h^{sb}, \qquad t_h = t_h^{sb} \equiv c_h q_h^{sb}.$$

Hence, we obtain the following familiar results. First, the optimal mechanism is deterministic ($q_h, q_l \in \{0, 1\}$). Second, the production decision of the efficient type, q_l^{sb}, is efficient ($C_l = c_l$), whilst the production decision of the inefficient type, q_h^{sb}, is distorted downwards ($C_h > c_h$). Finally, whenever the inefficient type has to produce, the efficient type obtains a positive rent. The inefficient type does not obtain a rent.

4 The Contracting Problem

We now turn to the problem when the agent observes the state s_2 after contracting. The problem of the principal is to design a contract (t, q) to maximize the value of production minus the transfer to the agent. The principal may use a general revelation mechanism to elicit the agent's private information.

Since the principal has full commitment, we may apply the revelation principle for multi–stage games of Myerson (1986) and concentrate on direct mechanisms. A direct mechanism is a combination $(t(\hat{s}_1, \hat{s}_2), q(\hat{s}_1, \hat{s}_2))$ which specifies transfers $t(\hat{s}_1, \hat{s}_2)$ to the agent and requires the agent to produce with probability $q(\hat{s}_1, \hat{s}_2)$ after the agent has submitted the two reports $\hat{s}_1 \in \{s_{1l}, s_{1h}\}$ and $\hat{s}_2 \in \{s_{2l}, s_{2h}\}$. We express a direct mechanism as $\{(t_{ij}, q_{ij})\}_{i,j\in\{l,h\}}$ with $(t_{ij}, q_{ij}) = (t(\hat{s}_{1i}, \hat{s}_{2j}), q(\hat{s}_{1i}, \hat{s}_{2j}))$. We stress an important feature of this dynamic setup: the revelation principle for multi–stage games demands that a report about s_1 should be submitted *before* the agent observes s_2. Thus, the direct mechanism has two reporting periods. A first period where the agent reports his initial observation s_1 and a second period where he reports s_2. Moreover, the direct mechanism must be incentive compatible along the equilibrium path.

As usual, the incentive constraints formalize the idea that the mechanism must give the agent an incentive to report his private information honestly. We start with considering the second period incentive constraints.

When the agent has to report his second observation s_2, he has already submitted his first report $\hat{s}_1$. This first report was either truthful or not. Importantly, the revelation principle for sequential games requires a truthful revelation of the second observation only for those cases in which the first report was truthful. It does *not* impose truthful revelation of information after a lie in the first stage.[3] This means that for each first period observation

[3] See also Myerson (1986, p.341). In this respect, our analysis differs from Courty and Li (2000) and Dai et al. (2008). In fact, these papers do not consider direct mechanisms in the strict sense of Myerson's multi–stage revelation principle. Instead they require that the agent reports his *final* cost type honestly in the second period. This approach is strategically equivalent to Myerson's multi–stage revelation principle, only if the support of final costs does not depend on the agent's ex ante private information. With a type–dependent support the complication arises that some final cost types cannot occur for certain ex ante information. In this case, the principal may do better by restricting the possible final cost

s_1, there are exactly *two* incentive constraints. In particular, for a low cost type who reported *truthfully* ($\hat{s}_1 = s_{1l}$), we have the two second period incentive constraints:

$$\begin{aligned} IC^2_{ll}: &\quad t_{ll} - c_{ll}q_{ll} \geq t_{lh} - c_{ll}q_{lh}, \\ IC^2_{lh}: &\quad t_{lh} - c_{lh}q_{lh} \geq t_{ll} - c_{lh}q_{ll}. \end{aligned} \tag{4}$$

Likewise, for a high cost type who reported *truthfully* ($\hat{s}_1 = s_{1h}$), we have the two second period incentive constraints:

$$\begin{aligned} IC^2_{hl}: &\quad t_{hl} - c_{hl}q_{hl} \geq t_{hh} - c_{hl}q_{hh}, \\ IC^2_{hh}: &\quad t_{hh} - c_{hh}q_{hh} \geq t_{hl} - c_{hh}q_{hl}. \end{aligned} \tag{5}$$

We now turn to the first period incentive constraint concerning the revelation of the agent's ex ante private information s_1. Since the agent has to report his observation without yet knowing the state s_2, the first period incentive constraints are non–standard. Suppose the agent observed the low state s_{1l}. As explained in the previous paragraphs, if the agent reports his observation honestly, the second period incentive constraints ensure that the agent also reports honestly in the second period. Hence, honestly reporting s_{1l} yields the utility

$$U^1_{ll} = \mu_l(t_{ll} - c_{ll}q_{ll}) + (1 - \mu_l)(t_{lh} - c_{lh}q_{lh}).$$

Instead of reporting honestly, the agent could lie and report a high state ($\hat{s}_1 = s_{1h}$). After lying the agent has to decide in period 2 whether to lie again or, this time, tell the truth. As said, the revelation principle does not impose a truthful revelation of information after a lie in the previous round. We therefore have to consider explicitly the possibility that after lying in the first stage, the contract induces the agent to lie again in the second stage.[4] Whether a low cost type who misreports his observation in the first stage will lie again or tell the truth in the second period, depends on which option yields the higher payoff. Hence, by announcing the high state in the first period, the low cost type can obtain the expected payoff

$$U^1_{lh} = \mu_l \max\{t_{hl} - c_{ll}q_{hl}, t_{hh} - c_{ll}q_{hh}\} + (1 - \mu_l) \max\{t_{hl} - c_{lh}q_{hl}, t_{hh} - c_{lh}q_{hh}\}.$$

types which the agent can report in the second period, because this reduces the agent's possible deviations. To avoid this complication of report-dependent message sets, we follow the direct approach of Myerson in our context with shifting supports.

[4]Indeed, we show that when α is large, then optimal contracts do induce lying off the equilibrium path.

We may therefore express the first period incentive constraint as

$$IC_l^1: \quad U_{ll}^1 \geq U_{lh}^1. \tag{6}$$

Likewise, a high cost type obtains the utility

$$U_{hh}^1 = \mu_h[t_{hl} - c_{hl}q_{hl}] + (1-\mu_h)(t_{hh} - c_{hh}q_{hh}),$$

when he truthfully reports his observation s_{1h}. By lying, the agent may obtain the utility

$$U_{hl}^1 = \mu_h \max\{t_{ll} - c_{hl}q_{ll}, t_{lh} - c_{hl}q_{lh}\} + (1-\mu_h)\max\{t_{lh} - c_{hh}q_{lh}, t_{ll} - c_{hh}q_{ll}\}.$$

The first period incentive constraint of the high cost type is

$$IC_h^1: \quad U_{hh}^1 \geq U_{hl}^1. \tag{7}$$

In order for the agent to participate, he has to obtain at least his type–independent reservation utility of zero.[5] That is, the mechanism must satisfy the following individual rationality constraints:

$$\begin{aligned} IR_l: &\quad U_{ll}^1 \geq 0, \\ IR_h: &\quad U_{hh}^1 \geq 0. \end{aligned} \tag{8}$$

We say that a direct mechanism $\{(t,q)\}$ is *incentive compatible* if it satisfies the incentive constraints (4)-(7). An incentive compatible direct mechanism is *feasible* when it satisfies the individual rationality constraints (8).

To summarize, the principal's problem is to find a feasible direct mechanism that maximizes the expected value of the agent's output minus the expected transfers:

$$\begin{aligned} P1: \max_{\{q,t\}} \quad W = &\quad \gamma[\mu_l V q_{ll} + (1-\mu_l)V q_{lh}] + (1-\gamma)[\mu_h V q_{hl} + (1-\mu_h)V q_{hh}] \\ &\quad -\gamma[\mu_l t_{ll} + (1-\mu_l)t_{lh}] - (1-\gamma)[\mu_h t_{hl} + (1-\mu_h)t_{hh}] \qquad (9) \\ \text{s.t.} &\quad (4),(5),(6),(7),(8). \end{aligned}$$

[5]If the principal found it optimal to exclude one type from the contract, she could achieve this by setting quantity and transfer for this type equal to zero. Therefore, it is without loss of generality to assume that the optimal contract is individually rational for both types.

5 Optimal Contracts

This section derives the optimal contract by solving the maximization problem $P1$ in a series of steps. We thereby will follow the standard analysis of monopolistic screening models as closely as possible. The following lemma takes the first step and identifies the individual rationality constraint of the inefficient type as the relevant one.

Lemma 1 *At the optimum the individual rationality constraint IR_h is binding. The individual rationality constraint IR_l follows from IR_h and IC_l^1.*

In the static model the analogue of Lemma 1 is a direct implication of the single–crossing property which guarantees that the agent's utility is increasing in his type. In the present sequential model, the agent's expected utility at the ex ante stage cannot be described by a single–crossing property. Instead, the correlation between states ($\mu_l \geq \mu_h$) implies that the agent's utility is increasing in his type.[6]

With the help of Lemma 1 we may show that, as usual, the relevant incentive constraint is the one of the efficient type, IC_l^1.

Lemma 2 *At the optimum the incentive compatibility constraint IC_l^1 is binding.*

Usually the incentive constraints imply an ordering on the quantity schedule q. Because in our framework only the second period incentive constraints are standard, incentive compatibility implies only a partial ordering.

Lemma 3 *The second period incentive constraints imply $q_{ll} \geq q_{lh}$ and $q_{hl} \geq q_{hh}$.*

The next lemma shows that the partial ordering implies that honesty also obtains off the equilibrium path where the low cost type lied about his first period observation and observes the second period cost s_{2l}.

Lemma 4 *An incentive compatible direct mechanism exhibits $t_{hl} - c_{ll}q_{hl} \geq t_{hh} - c_{ll}q_{hh}$.*

According to Lemma 2 the payoff of the low cost type, U_{ll}^1, equals U_{lh}^1 which, due to Lemma 4, simplifies to

$$U_{lh}^1 = \mu_l(t_{hl} - c_{ll}q_{hl}) + (1 - \mu_l)\max\{t_{hl} - c_{lh}q_{hl}, t_{hh} - c_{lh}q_{hh}\}.$$

[6]Courty and Li (2000) show that this result is generally true if the distribution of the ex post type conditional on the ex ante type can be ordered in terms of first or second order stochastic dominance. The positive correlation between states in our setup corresponds to first order stochastic dominance ordering. Dai et al. (2006) consider second order stochastic dominance.

This allows us to show that the incentive constraint IC^2_{hl} must bind at the optimum and, as a direct consequence, we may disregard the incentive constraint IC^2_{hh}.

Lemma 5 *The incentive constraint IC^2_{hl} binds at the optimum and IC^2_{hh} is slack.*

We may further use Lemma 5 to show that the out–of–equilibrium behavior of a low cost type, who reports a high first period cost, depends on the ordering of c_{hl} and c_{lh} and hence on α. In particular, when α is large, the optimal mechanism induces lying off the equilibrium path.

Lemma 6 *For $\alpha > \bar{\alpha}$ an optimal direct mechanism exhibits $t_{hl} - c_{lh}q_{hl} \geq t_{hh} - c_{lh}q_{hh}$. For $\alpha < \bar{\alpha}$ an optimal direct mechanism exhibits $t_{hl} - c_{lh}q_{hl} \leq t_{hh} - c_{lh}q_{hh}$.*

Following Lemma 6, we may simplify the payoff of the low cost type, U^1_{lh}, further as

$$U^1_{lh} = \begin{cases} \mu_l(t_{hl} - c_{ll}q_{hl}) + (1-\mu_l)(t_{hh} - c_{lh}q_{hh}) & \text{if } \alpha \leq \bar{\alpha} \\ \mu_l(t_{hl} - c_{ll}q_{hl}) + (1-\mu_l)(t_{hl} - c_{lh}q_{hl}) & \text{if } \alpha > \bar{\alpha}. \end{cases}$$

The previous lemmas identify IC^1_l, IR_h and IC^2_{hl} as the relevant constraints that are binding at the optimum. This suggests that we simplify the maximization problem to

$$\begin{aligned} P2: \quad \max_{\{q,t\}} \quad & \gamma[\mu_l V q_{ll} + (1-\mu_l)V q_{lh}] + (1-\gamma)[\mu_h V q_{hl} + (1-\mu_h)V q_{hh}] \\ & -\gamma[\mu_l t_{ll} + (1-\mu_l)t_{lh}] - (1-\gamma)[\mu_h t_{hl} + (1-\mu_h)t_{hh}] \qquad (10) \\ \text{s.t.} \quad & U^1_{ll} = U^1_{lh};\ U^1_{hh} = 0;\ t_{hl} - c_{hl}q_{hl} = t_{hh} - c_{hl}q_{hh} \\ & q_{ll} \geq q_{lh};\ q_{hl} \geq q_{hh}. \qquad (11) \end{aligned}$$

Compared to the original problem $P1$, the constraints IC^1_h, IC^2_{ll} and IC^2_{lh} are missing in $P2$. Our approach is to concentrate on $P2$ and show that its solution also solves $P1$. The next lemma shows that the constraints IC^2_{ll} and IC^2_{lh} do not restrict our solution, since we may always find a solution to program $P2$ that satisfies them.

Lemma 7 *We may assume without loss of generality that a solution to $P2$ exists for which the constraint IC^2_{lh} is satisfied in equality and IC^2_{ll} is slack.*

Lemma 7 implies that the only difference between $P1$ and $P2$ is that the solution to $P1$ is restricted by the additional constraint IC^1_h. In the standard monopolistic screening model, the corresponding incentive constraint for the inefficient type can be easily dealt with. It can be disregarded because it is automatically implied by the incentive constraint for the efficient type as a result of the single–crossing property and monotonicity of the allocation rule. This insight cannot be applied to the current sequential screening model since the incentive constraints in period 1 cannot be described in terms of monotonicity conditions. Our alternative approach is to derive the solution to $P2$ and then later verify directly that it satisfies IC_h^1.

Next, we solve $P2$. The two binding constraints IR_h and IC_{hl}^2 imply

$$t_{hl} = (1-\mu_h)(c_{hh}-c_{hl})q_{hh} + c_{hl}q_{hl}, \tag{12}$$
$$t_{hh} = (\mu_h c_{hl} + (1-\mu_h)c_{hh})q_{hh} = c_h q_{hh}. \tag{13}$$

With these expressions for t_{hl} and t_{hh} we may substitute out the constraints and express the principal's payoff independently of the transfers as

$$\begin{aligned} W &= \gamma[\mu_l(V-c_{ll})q_{ll} + (1-\mu_l)(V-c_{lh})q_{lh}] \\ &\quad +(1-\gamma)[\mu_h(V-c_{hl})q_{hl} + (1-\mu_h)(V-c_{hh})q_{hh}] - \gamma U_{lh}^1 \end{aligned} \tag{14}$$

with

$$U_{lh}^1 = \begin{cases} (c_h - c_l - \mu_l(c_{hl}-c_{ll}))q_{hh} + \mu_l(c_{hl}-c_{ll})q_{hl} & \text{if } \alpha \le \bar{\alpha} \\ (c_h - c_{hl})q_{hh} + (c_{hl}-c_l)q_{hl} & \text{if } \alpha > \bar{\alpha}. \end{cases}$$

As a consequence, solving the program $P2$ is identical to maximizing expression (14) with respect to q under the monotonicity conditions (11). Notice that the objective (14) equals total social surplus minus the information rent the principal needs to concede to the efficient type in order to meet the first period incentive compatibility constraint. Thus, the principal faces the familiar trade-off between efficiency and rent extraction.

It is helpful to disregard the monotonicity conditions in program $P2$ at this stage and compute the unconstrained optimal schedules q^u. If the unconstrained schedules satisfy the monotonicity conditions (11), then they represent a solution to the constrained problem $P2$ as well. Because expression (14) is linear in q, the optimal unconstrained schedule q_{ij}^u can be characterized by a threshold value C_{ij} as follows:

$$q_{ij}^u = \begin{cases} 1 & \text{if } V \ge C_{ij}, \\ 0 & \text{if } V < C_{ij}. \end{cases} \tag{15}$$

The values C_{ij} have the familiar interpretation as *virtual costs*. The virtual costs C_{lj} for the efficient type are equal to true costs: $C_{ll} \equiv c_{ll}$ and $C_{lh} \equiv c_{lh}$. In contrast, the virtual costs C_{hj} for the inefficient type exceed true costs:

$$C_{hl} = \begin{cases} c_{hl} + \frac{\gamma}{1-\gamma}\frac{\mu_l}{\mu_h}(c_{hl}-c_{ll}) & \text{if } \alpha \le \bar{\alpha}, \\ c_{hl} + \frac{\gamma}{1-\gamma}\frac{1}{\mu_h}(c_{hl}-c_l) & \text{if } \alpha > \bar{\alpha}, \end{cases}$$

and

$$C_{hh} = \begin{cases} c_{hh} + \frac{\gamma}{1-\gamma}\frac{1}{1-\mu_h}[c_h - c_l - \mu_l(c_{hl} - c_{ll})] & \text{if } \alpha \leq \bar{\alpha}, \\ c_{hh} + \frac{\gamma}{1-\gamma}\frac{1}{1-\mu_h}(c_h - c_{hl}) & \text{if } \alpha > \bar{\alpha}. \end{cases}$$

Does the unconstrained solution q^u also satisfy the second period monotonicity constraints $q_{ih} \leq q_{il}$? Notice first that monotonicity of the allocations is equivalent to $C_{il} \leq C_{ih}$. As for the efficient type $i = l$, we do have that $C_{ll} < C_{lh}$ so that indeed $q_{lh} \leq q_{ll}$. For the inefficient type $i = h$, the comparison of C_{hl} and C_{hh} depends on whether α is smaller or larger than $\bar{\alpha}$. We now define two threshold values for α at which C_{hl} equals C_{hh} depending on whether α is smaller or larger than $\bar{\alpha}$. Let $\Delta\mu = \mu_l - \mu_h$ and define

$$\alpha_1 \equiv 1 - \frac{\gamma\Delta\mu\Delta s_1}{\gamma\Delta\mu\Delta s_1 + \mu_h(1 - \mu_h - \gamma(1 - \mu_l))\Delta s_2},$$

and

$$\alpha_2 \equiv 1 - \frac{\gamma\Delta s_1}{\gamma\Delta s_1 + (\gamma(1 - \mu_l) + \mu_h)\Delta s_2}.$$

It is straightforward to verify that whenever $\gamma\mu_l > \mu_h$, we have $\alpha_1 < \alpha_2 < \bar{\alpha}$, and whenever $\gamma\mu_l < \mu_h$, we have $\bar{\alpha} < \alpha_2 < \alpha_1$. The next lemma gives a full characterization of the optimal schedules q^* for program $P2$.

Lemma 8 *Whenever $\alpha \leq \min\{\alpha_1, \alpha_2\}$ the solution q^u as defined in (15) satisfies the monotonicity constraints and represents an optimal schedule for program $P2$, $q^* = q^u$. Whenever $\alpha > \min\{\alpha_1, \alpha_2\}$, the optimal solution to $P2$ exhibits $q^*_{ll} = q^u_{ll}$; $q^*_{lh} = q^u_{lh}$; and bunching $q^*_{hl} = q^*_{hh} = q^{sb}_h$.*

The lemma shows that bunching concerning the allocation q_{hl} and q_{hh} is an issue exactly when the degree of ex post private information is small. Intuitively, as α becomes large, the efficient type's incentive to lie off the equilibrium path increases. In fact, Lemma 6 demonstrated that if $\alpha > \bar{\alpha}$, the efficient type does lie off the equilibrium path. In this case, the rent U^1_{lh} conceded to the efficient type does not directly depend on the allocation q_{hh}, but only on q_{hl}. Hence, in order to reduce the rent the principal has a stronger incentive for a downward distortion of the allocation q_{hl} than for distorting the allocation q_{hh}. Yet, this tendency conflicts with the monotonicity requirement $q_{hl} \geq q_{hh}$ and leads to bunching.

Having solved problem $P2$, we can now return to the original problem $P1$. Recall that the only difference between the two problems is that $P1$ includes the additional constraint IC^1_h. The next proposition demonstrates, however, that the solution to $P2$ automatically satisfies IC^1_h and is thus also a solution to $P1$.

Proposition 1 *The solution q^* of Lemma 8 represents an optimal schedule for program $P1$.*

We summarize the most salient features of the optimal contract in a corollary.

Corollary 1 *(i) There is no distortion at the top and a downward distortion at the bottom, that is, q^*_{ll} and q^*_{lh} are efficient, and q^*_{hl} and q^*_{hh} are below the efficient quantity.*
(ii) When the degree of ex post private information is high ($\alpha < \min\{\alpha_1, \alpha_2\}$), then for the range of values $V \in (C_{hl}, C_{hh})$, the allocations for the inefficient type are strictly ordered: $q^*_{hh} < q^*_{hl}$.
(iii) When the degree of ex post private information is low ($\alpha > \min\{\alpha_1, \alpha_2\}$), then for all values V, the allocation for the inefficient type coincides with the second best solution without ex post private information: $q^*_{hl} = q^*_{hh} = q^{sb}_h$.
(iv) Whenever the states s_1 and s_2 are correlated ($\mu_h \neq \mu_l$) or there is untruthful reporting off the equilibrium path ($\alpha > \bar{\alpha}$), the optimal allocation depends on the distribution of the state s_2.
(v) If $\alpha = 0$, the first best is obtained if and only if there is no correlation ($\mu_l = \mu_h$). If $\alpha = 1$, the second best solution of Section 3 obtains irrespective of μ_h and μ_l.

Part (i) is a well–known property and is consistent with the analysis of Courty and Li (2000) and Battaglini (2005). No distortion at the top arises because the information rent conceded to the efficient type is independent of the quantities q_{ll} and q_{lh}. Further, Courty and Li (2000) have shown that downward distortions at the bottom emerge in sequential screening if the distribution of the ex post types conditional on the ex ante types can be ordered in terms of first order stochastic dominance. Our assumption that states are positively correlated is a special case of such an ordering.

Part (ii) describes the allocations for the inefficient type when there is no bunching. If there is no bunching, the virtual costs satisfy $C_{hl} < C_{hh}$, and this implies the ordering $0 = q^*_{hh} < q^*_{hl} = 1$ whenever $V \in (C_{hl}, C_{hh})$.

Part (iii) demonstrates a strong robustness result of the monopolistic screening model: ex post private information does not affect the allocation for the inefficient type when the degree of ex post private information is small. As explained above, this result is due to bunching and does not appear in Courty and Li (2000) or Battaglini (2005), because they assume that the support of total costs is independent of the state s_1. This implies that lying off the equilibrium path and thus bunching never occurs.

In addition, Courty and Li (2000, p.711) argue that in the additive sequential screening model the optimal allocation is independent of the distribution of the state s_2. Part (iv) of the observation reveals that this result depends on 1) the absence of correlation between states and 2) truthful reporting off the equilibrium path ($\alpha < \bar{\alpha}$).[7]

Finally, the intuition behind part (v) is that, when states are correlated, the observation s_1 is informative about the agent's expected total costs, and thus, the agent possesses relevant ex ante private information even for $\alpha = 0$. Without correlation ($\mu_l = \mu_h$) the agent does not have any ex ante private information and the optimal contract is first best. At the other extreme $\alpha = 1$, the state s_2 is irrelevant for costs, and since $\alpha = 1 > \min\{\alpha_1, \alpha_2\}$ we get the second best solution when there is only ex ante private information.

6 The Effect Of Ex Post Private Information

By contrasting the outcome of Proposition 1 to the benchmark outcome in Section 3, we now investigate the implications of the presence of ex post information on the distribution of rents and efficiency.

We begin with an observation for the case in which the degree of ex post private information is small ($\alpha > \min\{\alpha_1, \alpha_2\}$). In this case, there is bunching and the allocation for the inefficient type is the same both with and without ex post information. As a result, the information rent for the efficient type is the same in both cases. Moreover, overall efficiency unambiguously increases with ex post information since the new information is exclusively used to adapt the efficient type's allocation to the new cost circumstances in a first best manner.

In contrast, the effects are less straightforward if the degree of ex post private information becomes more significant. To demonstrate the diverse, ambiguous effects, we focus on the case where ex post private information plays an intermediate role ($\bar{\alpha} < \alpha < \alpha_2$). In this case we have the ordering $c_{ll} < c_l < c_{hl} < C_{hl} < c_{lh} < c_h < C_h < c_{hh} < C_{hh}$. Figure 1 illustrates how the equilibrium outcome changes as a function of the project's value V with and without ex post information.

Information rents

We first discuss the effect of ex post private information on information rents.[8] With ex post information, the efficient type who has observed the state s_{1l} receives an information rent of U_{lh}. Without ex post information his rent

[7] To see this, note that if $\mu_h = \mu_l$, then virtual costs C_{hl} and C_{hh} depend on μ_h if and only if $\alpha > \bar{\alpha}$.

[8] Note that the principal's payoff is weakly larger with ex post than without ex post information. This is so since the second best contract without ex post information is feasible also in the presence of ex post information.

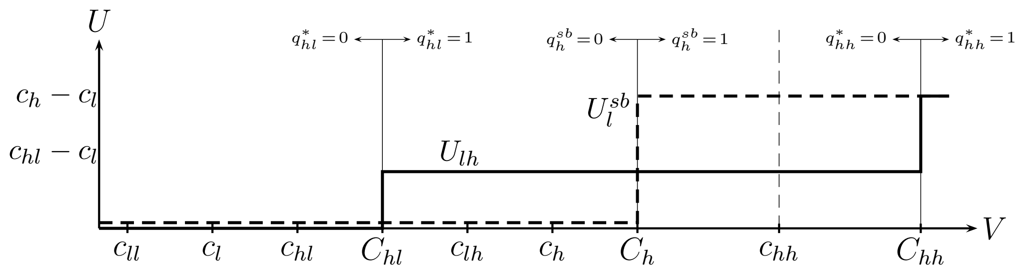

Figure 1: Payoffs for $\bar{\alpha} < \alpha < \alpha_2$

equals $U_l^{sb} = (c_h - c_l)q_h^{sb}$. We therefore define

$$\Delta U \equiv U_{lh} - U_l^{sb} \tag{16}$$

as the difference in information rents due to ex post private information. In Figure 1, ΔU is the difference between the solid and the dashed line. The figure shows that, depending on the parameter constellation, ΔU may be positive or negative. Moreover, the arrows at the top of Figure 1 indicate the allocations for the inefficient cost type ($s_1 = s_{1h}$) in the optimal contract both with and without ex post information.

To understand the intuition behind the sign of ΔU, recall that for a given contract the information rent for the efficient type is lower the lower the probability with which the inefficient type has to produce. This is so since the lower the production probability for the high cost type, the smaller is the extent to which the low cost type could cash in on cost advantages by artificially exaggerating his costs.

Observation 1 *For $V < C_{hl}$, ex post private information does not affect the agent's equilibrium payoff.*

Irrespective of the presence of ex post private information, the agent does not receive any rents. This is because for $V < C_{hl}$ the optimal contract in both the absence and the presence of ex post information does not induce a high cost type to produce. Hence, only a low cost type produces, and the principal does not have to pay this type an information rent to prevent him from pretending to be a high cost type.

Observation 2 *For $V \in [C_{hl}, C_h]$, the agent's payoff is larger with ex post private information.*

The intuition behind this result is that the optimal contract with ex post information induces production for a high cost type if he obtains favorable ex post information about s_2 ($q^*_{hl} = 1$) while the optimal contract without ex post information never induces production for a high cost type ($q^{sb}_h = 0$). Hence, only with ex post private information the principal must pay the low cost type a strictly positive information rent in order to prevent him from claiming to be a high cost type.

Observation 3 *For $V \in [C_h, C_{hh})$, ex post private information hurts the agent.*

This occurs because without ex post private information the principal allows a high cost type to produce with probability one ($q^{sb}_h = 1$), while with ex post information a high cost type is allowed to execute the project only if its second observation is favorable ($q^*_{hh} = 0$). Thus, the low cost type has a relatively stronger incentive to imitate the high cost type in the absence than in the presence of ex post information. Consequently, the information rent that is required to prevent the low cost type from mimicking a high cost type is lower in the presence of ex post information. Hence, we obtain the somewhat counter intuitive result that the agent's ex post private information actually hurts him.

Finally, for $V \geq C_{hh}$ the implementation decision, and therefore the information rent, is independent of ex post information. In sum, the previous considerations show that the effect of ex post private information on information rents is generally ambiguous. Moreover, the difference in the agent's rent, ΔU, is non–monotonic in the project's value.

Efficiency

Next we address how ex post private information affects economic distortions. Note that from a welfare perspective, the project should be executed if and only if the project's value V exceeds the project's cost c. Hence, more accurate information about costs permits, in principle, a more efficient implementation decision. For instance, for $V \in (c_{hl}, c_h)$ ex post information about s_2 is socially valuable, because in this case it is efficient to implement the project when the second observation reveals that costs equal c_{hl}. In contrast, absent ex post information, it is efficient to cancel the project since it is only known that expected costs are $c_h > V$.

The next observations demonstrate that the principal may not use ex post information efficiently under the optimal contract. As a result, ex post information might decrease overall welfare. To make this point, we compare the first best allocation that arises if information is public with the allocations implemented under the optimal contract with and without ex post information.

We display these allocations in tables of the following sort:

	$s_2 = s_{2l}$	$s_2 = s_{2h}$
$s_1 = s_{1l}$	$q_{ll}^{fb}, q_l^{sb}, q_{ll}^*$	$q_{lh}^{fb}, q_l^{sb}, q_{lh}^*$
$s_1 = s_{1h}$	$q_{hl}^{fb}, q_h^{sb}, q_{hl}^*$	$q_{hh}^{fb}, q_h^{sb}, q_{hh}^*$

Table 1: First best, second best, and optimal allocations.

For example, the top left cell depicts the first best allocation q_{ll}^{fb} and the allocations under the optimal contract without (q_l^{sb}) and with (q_{ll}^*) ex post information when both the states s_1 and s_2 are low so that total cost is c_{ll}. The next tables exhibit these allocations for the parameter range $V \in (c_{hl}, C_{hh})$:

$1,1,1$	$0,1,0$
$1,0,0$	$0,0,0$

$V \in (c_{hl}, C_{hl})$

$1,1,1$	$0,1,0$
$1,0,1$	$0,0,0$

$V \in (C_{hl}, c_{lh})$

$1,1,1$	$1,1,1$
$1,0,1$	$0,0,0$

$V \in (c_{lh}, C_h)$

$1,1,1$	$1,1,1$
$1,1,1$	$0,1,0$

$V \in (c_h, c_{hh})$

$1,1,1$	$1,1,1$
$1,1,1$	$1,1,0$

$V \in (c_{hh}, C_{hh})$

Figure 2: Allocations as a function of V.

The first set of tables in the top line of Figure 2 covers the range $V \in (c_{hl}, c_{hh})$. Inspection reveals that the contract with ex post information implements the first best allocation or the same allocation as the contract without ex post information. In fact, there is always at least one constellation of costs in which the contract with ex post information implements the first best while the contract without ex post information implements an inefficient allocation. Thus, welfare strictly increases in the presence of ex post information:

Observation 4 *If $V \in (c_{hl}, c_{hh})$, then ex post private information strictly increases welfare.*

The reason why welfare is increased is that the additional information helps to better fine–tune the allocation to the prevailing cost circumstances. This does not imply, however, that information is efficiently used under the optimal contract with ex post information. To see this, consider the first table and consider the case in which the cost is c_{hl}. In this case it is efficient to implement the allocation $q_{hl}^{fb} = 1$, since $c_{hl} < V$. However, the optimal contract implements the inefficient allocation $q_{hl}^{*} = 0$. The reason is that setting $q_{hl} = 1$ would require the principal to pay an information rent to the agent which exceeds the increase in the aggregate surplus and therefore lowers the principal's own payoffs. This demonstrates that, due to information rents, ex post private information is not always used efficiently.

Even though information might be used inefficiently, ex post information does not worsen distortions if $V \in (c_{hl}, c_{hh})$. One of our key findings is that this does not hold generally. As displayed in the final table of Figure 2 welfare is lower with ex post private information when $V \in (c_{hh}, C_{hh})$:

Observation 5 *For $V \in (c_{hh}, C_{hh})$, ex post private information decreases welfare.*

The reasoning behind this results is that, for $V > c_{hh}$, it is efficient to implement the project for any cost realization. The optimal contract without ex post private information does indeed implement this efficient allocation ($q_l^{sb} = q_h^{sb} = 1$). In contrast, the optimal contract with ex post private information does not. It leads to a cancelation of the project ($q_{hh} = 0$) when costs equal c_{hh}. Consequently, for $V \in (c_{hh}, C_{hh})$ ex post private information leads to lower overall efficiency. The fact that the principal finds it optimal to induce a less efficient allocation is once again due to her concerns about information rents. In this case, however, her concerns have the perverse effect that the agent loses more information rents than the principal gains. This leads to a reduction in overall welfare and efficiency.

7 Conclusion

We investigate the role of ex post private information in the monopolistic screening model. We show that, due to bunching, small degrees of ex post private information do not affect distortions. This represents a strong robustness result for the monopolistic screening model. For larger degrees of ex post private information, however, effects are non–trivial and we obtain ambiguous results on the agent's equilibrium payoffs and overall distortions. Due to the principal's concerns about information rents, the agent may lose or gain from his additional private information. More importantly, economic distortions may increase or decrease. These insights demonstrate that the effects of additional private information are more subtle than superficial intuition might suggest.

Our results have implications for settings where regulators are concerned with promoting allocative efficiency. For instance, in firm–to–firm procurement the question arises whether procurement contracts should be allowed to include terms that condition on future developments that affect production costs. The present work shows that this should not be the case when this revelation of information serves only the purpose of reducing information rents. Our analysis also bears on the discussion of disclosure duties of agents in fiduciary relationships. Here the question is to what extent the agent has a duty in revealing new information to his principal. Our analysis suggests that this is indeed beneficial to the principal, but it may be at the expense of allocative efficiency. Moreover, our results are relevant for the discussions of confidentiality and privacy rules that are meant to protect agents from too much disclosure. These rules may be understood as means to restrict the harmful effects of the disclosure of ex post private information. Yet, our analysis also calls for caution, as the ultimate effects of ex post private information are subtle and depend crucially on the underlying context. It provides only a broad intuition for assessing the likely effect: whenever information rents play an important role, disclosure of ex post information may harm allocative efficiency.

Appendix

Proof of Lemma 1: We show that IR_l follows from IR_h and IC_l^1. Indeed,

$$\begin{aligned} & \mu_l[t_{ll} - c_{ll}q_{ll}] + (1-\mu_l)(t_{lh} - c_{lh}q_{lh}) \\ \geq\ & \mu_l[t_{hl} - c_{ll}q_{hl}] + (1-\mu_l)(t_{hh} - c_{lh}q_{hh}) \\ \geq\ & \mu_l[t_{hl} - c_{hl}q_{hl}] + (1-\mu_l)(t_{hh} - c_{hh}q_{hh}) \\ \geq\ & \mu_h[t_{hl} - c_{hl}q_{hl}] + (1-\mu_h)(t_{hh} - c_{hh}q_{hh}) \geq 0. \end{aligned}$$

The first inequality follows from IC_l^1, the second inequality uses $c_{ll} < c_{hl}$ and $c_{lh} < c_{hh}$, the third inequality follows from $\mu_l \geq \mu_h$ together with $t_{hl} - c_{hl}q_{hl} \geq t_{hh} - c_{hl}q_{hh} \geq t_{hh} - c_{hh}q_{hh}$, the final inequality follows from IR_h. Hence, IR_l does not bind at the optimum. Yet, at the optimum at least one individual rationality constraint must be binding since otherwise the principal could increase his payoff by lowering all transfers by some $\varepsilon > 0$. Hence, IR_h must be binding. Q.E.D.

Proof of Lemma 2: Suppose we find an optimal mechanism for which IC_l^1 does not bind, then we may lower both transfers t_{lh} and t_{ll} by $\varepsilon > 0$ such that IC_l^1 remains satisfied. Since this alternative mechanism continues to fulfill IR_h, Lemma 1 implies that the alternative mechanism also satisfies IR_l. Also all other constraints remain unaffected so that the alternative mechanism is also feasible. But since it yields the principal a larger payoff, we arrive at the contradiction that the original mechanism could not have been optimal. Q.E.D.

Proof of Lemma 3: Adding the constraints IC_{il}^2 and IC_{ih}^2, $i = l, h$, yields

$$
\begin{aligned}
& t_{ih} - c_{ih}q_{ih} + t_{il} - c_{il}q_{il} \geq t_{il} - c_{ih}q_{il} + t_{ih} - c_{il}q_{ih} \\
\Rightarrow \quad & (c_{ih} - c_{il})q_{il} \geq (c_{ih} - c_{il})q_{ih} \\
\Rightarrow \quad & (c_{ih} - c_{il})(q_{il} - q_{ih}) \geq 0.
\end{aligned}
$$

Since $c_{ih} > c_{il}$ we must have $q_{il} \geq q_{ih}$ for the product to be non–negative. Q.E.D.

Proof of Lemma 4: We have

$$t_{hl} - q_{hl}c_{ll} - t_{hh} + q_{hh}c_{ll} = t_{hl} - t_{hh} - c_{ll}(q_{hl} - q_{hh}) \geq t_{hl} - t_{hh} - c_{hl}(q_{hl} - q_{hh}) \geq 0,$$

where the first inequality follows from $q_{hl} \geq q_{hh}$ together with $c_{ll} < c_{hl}$ and the second inequality follows from the incentive constraint IC_{hl}^2. Q.E.D.

Proof of Lemma 5: Suppose that we have an optimal mechanism such that IC_{hl}^2 is slack. That is,

$$\delta = t_{hl} - c_{hl}q_{hl} - (t_{hh} - c_{hl}q_{hh}) > 0.$$

From this mechanism we may construct an alternative mechanism and lower t_{hl} by adding $\Delta t_{hl} = -(1 - \mu_h)\delta$ and raise t_{hh} by adding $\Delta t_{hh} = \mu_h\delta$. The changed transfers leave the expected transfer to the high cost type unchanged: $\mu_h\Delta t_{hl} + (1-\mu_h)\Delta t_{hh} = 0$. The change makes IC_{hl}^2 binding, since $\Delta t_{hl} - \Delta t_{hh} = -\delta$. It also relaxes the incentive constraint IC_l^1, because it changes the right hand side of IC_l^1 by

$$\mu_l\Delta t_{hl} + (1 - \mu_l)\Delta t_{hh} < \mu_h\Delta t_{hl} + (1 - \mu_h)\Delta t_{hh} = 0.$$

This implies that under the alternative mechanism, IC_l^1 is slack.

Since the change in transfers does not affect the other incentive constraints, the alternative mechanism is feasible whenever the original mechanism is feasible. Since it is payoff equivalent to the original mechanism, it must also be optimal if the original mechanism is optimal. Applying Lemma 2, we obtain the contradiction that the alternative mechanism cannot be optimal, because, by construction, IC_l^1 is not binding. As a consequence, a mechanism for which IC_{hl}^2 is slack cannot be optimal.

Moreover, a binding IC_{hl}^2 implies $t_{hl} - t_{hh} = c_{hl}(q_{hl} - q_{hh})$, so that $t_{hh} - c_{hh}q_{hh} - (t_{hl} - c_{hh}q_{hl}) = (c_{hh} - c_{hl})(q_{hl} - q_{hh}) \geq 0$, where the inequality follows from $q_{hl} \geq q_{hh}$ and $c_{hh} > c_{hl}$. Q.E.D.

Proof of Lemma 6: A binding IC_{hl}^2 implies $t_{hl} - t_{hh} = c_{hl}(q_{hl} - q_{hh})$, so that we have

$$t_{hl} - c_{lh}q_{hl} - (t_{hh} - c_{lh}q_{hh}) = c_{hl}(q_{hl} - q_{hh}) - c_{lh}(q_{hl} - q_{hh}) = (c_{hl} - c_{lh})(q_{hl} - q_{hh}).$$

Because $q_{hl} \geq q_{hh}$ the expression can only be positive if $c_{hl} > c_{lh}$ which is the case exactly when $\alpha > \bar{\alpha}$. Otherwise the expression is non–positive Q.E.D.

Proof of Lemma 7: We show that for any output schedule $\bar{q} = (\bar{q}_{ll}, \bar{q}_{lh}, \bar{q}_{hl}, \bar{q}_{hh})$ and transfers $(\bar{t}_{ll}, \bar{t}_{lh}, \bar{t}_{hl}, \bar{t}_{hh})$ that satisfy the three equality constraints in $P2$, we may find a payoff equivalent combination $\bar{q}$ and $(t'_{ll}, t'_{lh}, \bar{t}_{hl}, \bar{t}_{hh})$ that also satisfy the three equality constraints and, in addition, satisfies the constraint IC_{lh}^2 in equality. In particular, define

$$\begin{aligned} t'_{ll} &= (1-\mu_l)\bar{t}_{lh} + \mu_l\bar{t}_{ll} + (1-\mu_l)(\bar{q}_{ll} - \bar{q}_{lh})c_{lh}, \\ t'_{lh} &= (1-\mu_l)\bar{t}_{lh} + \mu_l\bar{t}_{ll} - \mu_l(\bar{q}_{ll} - \bar{q}_{lh})c_{lh}. \end{aligned}$$

Since in the modified mechanism only the transfers to the low type are changed, it obviously satisfies the constraints $U_{hh}^1 = 0$ and $t'_{hl} - c_{hl}q_{hl} = t'_{hh} - c_{hl}q_{hh}$. Further, the expected transfer to the low type is the same under the old and the modified mechanism:

$$\mu_l\bar{t}_{ll} + (1-\mu_l)\bar{t}_{lh} = \mu_l t'_{ll} + (1-\mu_l)t'_{lh}.$$

This implies that the modified mechanism also maintains to satisfy the constraint $U_{ll}^1 = U_{lh}^1$ and is payoff equivalent to the original mechanism.

Finally, note that when IC_{lh}^2 is binding, then $q_{ll} \geq q_{lh}$ implies that IC_{ll}^2 is automatically satisfied:

$$t_{ll} - t_{lh} - c_{ll}q_{ll} - c_{ll}q_{lh} \geq t_{ll} - t_{lh} - c_{lh}(q_{ll} - q_{lh}) = 0.$$

Q.E.D.

Proof of Lemma 8: Straightforward calculations show that for $\alpha \leq \bar{\alpha}$ we have $q_{hh}^u \leq q_{hl}^u$ if and only if $\alpha < \alpha_1$.

Likewise, for $\alpha > \bar{\alpha}$ we obtain $q^u_{hh} \leq q^u_{hl}$ if and only if $\alpha < \alpha_2$. Whenever $\gamma\mu_l > \mu_h$ we have $\alpha_1 < \alpha_2 < \bar{\alpha}$ and whenever, $\gamma\mu_l < \mu_h$ it holds $\bar{\alpha} < \alpha_2 < \alpha_1$. From these latter observations it follows that the solution q^u satisfies the monotonicity condition (11) exactly when $\alpha \leq \min\{\alpha_1, \alpha_2\}$. In other case, $\alpha > \min\{\alpha_1, \alpha_2\}$, the relaxed solution exhibits $q^u hl < q^u_{hh}$ and the solution to $P2$ exhibits bunching with $q_{hl} = q_{hh} = q^{sb}_h$. Q.E.D.

Proof of Proposition 1: To simplify notation, write plain q for the optimal quantity schedule q^* given in Lemma 8. We have to show that optimal mechanism satisfies $U^1_{hh} \geq U^1_{hl}$. We first prove two claims by which we may simplify the maximum expressions in U^1_{hl}.

- *Claim 1*: We have $t_{lh} - c_{hh}q_{lh} \geq t_{ll} - c_{hh}q_{ll}$.
  Indeed,

$$t_{lh} - c_{hh}q_{lh} - [t_{ll} - c_{hh}q_{ll}] = (t_{lh} - t_{ll}) - c_{hh}(q_{lh} - q_{ll}) \geq (t_{lh} - t_{ll}) - c_{lh}(q_{lh} - q_{ll}) \geq 0,$$

where the last inequality follows from IC^2_{lh}. This proves Claim 1.

- *Claim 2*: If IC^2_{lh} binds we have $t_{ll} - c_{hl}q_{ll} \geq t_{lh} - c_{hl}q_{lh} \;\Leftrightarrow c_{lh} \geq c_{hl}$.
  Indeed, if IC^2_{lh} binds we have

$$\begin{aligned} t_{ll} - c_{hl}q_{ll} - (t_{lh} - c_{hl}q_{lh}) &= t_{ll} - t_{lh} - c_{hl}(q_{ll} - q_{lh}) \\ &= c_{lh}(q_{ll} - q_{lh}) - c_{hl}(q_{ll} - q_{lh}) \\ &= (c_{lh} - c_{hl})(q_{ll} - q_{lh}). \end{aligned}$$

Hence, if $c_{lh} \geq c_{hl}$ we have $t_{ll} - c_{hl}q_{ll} \geq (t_{lh} - c_{hl}q_{lh})$. If $c_{lh} \leq c_{hl}$ we have $t_{ll} - c_{hl}q_{ll} \leq (t_{lh} - c_{hl}q_{lh})$, and this establishes Claim 2.

Claim 1 implies that the maximum expression in the first term of U^1_{hl} always equals $t_{lh} - c_{hh}q_{lh}$. Claim 2 implies that to expand the maximum expression in the second term of U^1_{hl} we have to distinguish two cases:

- *Case 1*: $c_{lh} \geq c_{hl}$: Claim 1 and 2 imply that we can write $U^1_{hh} - U^1_{hl}$ as

$$\begin{gathered} \mu_h(t_{hl} - c_{hl}q_{hl}) + (1 - \mu_h)(t_{hh} - c_{hh}q_{hh}) \\ -\mu_h(t_{ll} - c_{hl}q_{ll}) - (1 - \mu_h)(t_{lh} - c_{hh}q_{lh}). \end{gathered}$$

We show that this is non–negative: Since IC^1_l is binding, we can add the left and subtract the right hand side of IC^1_l so that $U^1_{hh} - U^1_{hl}$ becomes

$$\begin{gathered} \mu_h(t_{hl} - c_{hl}q_{hl}) + (1 - \mu_h)(t_{hh} - c_{hh}q_{hh}) - \mu_h(t_{ll} - c_{hl}q_{ll}) - (1 - \mu_h)(t_{lh} - c_{hh}q_{lh}) \\ +\mu_l(t_{ll} - c_{ll}q_{ll}) + (1 - \mu_l)(t_{lh} - c_{lh}q_{lh}) - \mu_l(t_{hl} - c_{ll}q_{hl}) - (1 - \mu_l)(t_{hh} - c_{lh}q_{hh}). \end{gathered}$$

The transfers in this expression simplify to $\Delta\mu[(t_{ll} - t_{lh}) + (t_{hh} - t_{hl})]$, where $\Delta\mu \equiv \mu_l - \mu_h \geq 0$. Since IC^2_{lh} and IC^2_{hl} are binding, we have $t_{ll} - t_{lh} = c_{lh}(q_{ll} - q_{lh})$ and $t_{hh} - t_{hl} = c_{hl}(q_{hh} - q_{hl})$. With an additional rearrangement of the cost terms in the above expression, we obtain

$$\begin{aligned} U^1_{hh} - U^1_{hl} &= \Delta\mu[c_{lh}(q_{ll} - q_{lh}) + c_{hl}(q_{hh} - q_{hl})] \\ &\quad +\Delta\mu[-c_{ll}(q_{ll} - q_{lh}) + -c_{hh}(q_{hh} - q_{hl}) + (c_{hh} - c_{ll})(q_{lh} - q_{hl})] \\ &\quad +\mu_h(c_{hl} - c_{ll})(q_{ll} - q_{hl}) + (1 - \mu_l)(c_{hh} - c_{lh})(q_{lh} - q_{hh}). \end{aligned}$$

Finally, if we unite the square brackets and once more rearrange, we end up with

$$\begin{aligned} U^1_{hh} - U^1_{hl} = \Delta\mu[&(c_{hh} - c_{lh})(q_{lh} - q_{hh}) + (c_{lh} - c_{hl})(q_{ll} - q_{hh}) + (c_{hl} - c_{ll})(q_{ll} - q_{hl})] \\ &+\mu_h(c_{hl} - c_{ll})(q_{ll} - q_{hl}) + (1 - \mu_l)(c_{hh} - c_{lh})(q_{lh} - q_{hh}). \end{aligned}$$

Now observe that all brackets in this expression are non–negative. For the brackets containing cost terms this follows by assumption. To see the claim for the brackets containing q-terms, recall that at the optimal solution we have for $i = l, h$ that $q_{li} = q^{fb}_{li}$ and $q_{hi} < q^{fb}_{hi}$. Thus, $c_{lh} < c_{hh}$ implies that $q_{lh} = q^{fb}_{lh} > q^{fb}_{hh} > q_{hh}$, which shows that, e.g. the first bracket $(q_{lh} - q_{hh})$ is positive. The argument for the other brackets is analogous. This establishes that IC^1_h is satisfied in Case 1.

- *Case 2*: $c_{lh} \geq c_{hl}$: Claim 1 and 2 imply that we can write $U^1_{hh} - U^1_{hl}$ as

$$\mu_h(t_{hl} - c_{hl}q_{hl}) + (1 - \mu_h)(t_{hh} - c_{hh}q_{hh}) - \mu_h(t_{lh} - c_{hl}q_{lh}) - (1 - \mu_h)(t_{lh} - c_{hh}q_{lh}).$$

As in Case 1, we add and subtract the two sides of IC^1_l and obtain after a rearrangement of terms:

$$\begin{aligned} U^1_{hh} - U^1_{hl} &= \Delta\mu[t_{ll} - t_{lh} - c_{ll}(q_{ll} - q_{lh}) + t_{hh} - t_{hl} - c_{hh}(q_{hh} - q_{hl}) \\ &\qquad +(c_{hh} - c_{ll})(q_{lh} - q_{hl})] \\ &\quad +\mu_h(t_{ll} - t_{lh} - c_{ll}(q_{ll} - q_{lh})) \\ &\quad +(1 - \mu_l)(t_{hh} - t_{hl} - c_{hh}(q_{hh} - q_{hl})) \\ &\quad +\mu_h(c_{hl} - c_{ll})(q_{lh} - q_{hl}) + (1 - \mu_l)(c_{hh} - c_{lh})(q_{lh} - q_{hl}). \end{aligned}$$

Using the binding constraints IC^2_{hl} and IC^2_{lh} yields:

$$\begin{aligned} U^1_{hh} - U^1_{hl} &= \Delta\mu[(c_{lh} - c_{ll})(q_{ll} - q_{lh}) + (c_{hl} - c_{hh})(q_{hh} - q_{hl}) \\ &\qquad +(c_{hh} - c_{ll})(q_{lh} - q_{hl})] \\ &\quad +\mu_h(c_{lh} - c_{ll})(q_{ll} - q_{lh}) + (1 - \mu_l)(c_{hl} - c_{hh})(q_{hh} - q_{hl}) \\ &\quad +\mu_h(c_{hl} - c_{ll})(q_{lh} - q_{hl}) + (1 - \mu_l)(c_{hh} - c_{lh})(q_{lh} - q_{hl}). \end{aligned}$$

Identical arguments as those used in Case 1 imply that the first three lines are non–negative. The fourth line is non–negative if $q_{lh} - q_{hl} \geq 0$. To see this, note that because $c_{lh} < c_{hl}$ we have $q_{lh} = q_{lh}^{fb} > q_{hl}^{fb} > q_{hl}$. This establishes that IC_h^1 is also satisfied in Case 2 and completes the proof. Q.E.D.

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